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03/11/21

Q1.) i.) $2x + 4y - 5z - 5x - 9y + 2z + 4x - 7y + 8z$

(5)

~~100~~

~~$x - 12y + 7z$~~

$x - 12y + 5z$

ii.) $x(x+2) - (x-4)$

$x^2 + 2x - x + 4$

$x^2 + x + 4$

(5)

iii.) $3p^2qr \times 4p^2q^3 \times 5qr^2$

$60p^3q^5r^3$

(5)

iv.) $\frac{6p^2q^3r}{3p^3q^3r^2} = 2 \left(\frac{p^2q^3rr^2}{p^3q^3} \right) = \frac{2r^3}{p}$

(5)

v.) $\frac{2fg}{16h} \times \frac{4gh^2}{4fh} \times \frac{32fh^3}{12f^3}$

$= \frac{256}{768} \left(\frac{f^2g^2h^5}{f^4h^2} \right)$

$= \frac{1}{3} \frac{g^2h^3}{f^2}$

$= \frac{g^2h^3}{3f^2}$

(5)

Q2) i.) $5(x-3) = 2(x+6)$

$$5x - 15 = 2x + 12$$

$$3x = 27$$

$$\underline{\underline{x = 9}}$$

(10)

ii.) ~~$x(x+2) - (x-4)$~~

~~$x^2 + 2x - x + 4$~~

~~$x^2 + x + 4$~~

$$\frac{(x+6)}{2} = x + \frac{(2x-5)}{3}$$

$$3(x+6) = 6x + 2(2x-5)$$

$$3x + 18 = 6x + 4x - 10$$

$$3x + 18 = 10x - 10$$

$$28 = 7x$$

$$\underline{\underline{4 = x}}$$

(10)

3.)

$$2x + 4y = 16 \quad (1)$$

$$3x + 2y = 7 \quad (2)$$

$$(1) - 2(2):$$

$$\begin{array}{r} 2x + 4y = 16 \\ - 6x + 4y = 14 \\ \hline -4x = 2 \end{array}$$

$$x = -\frac{1}{2}$$

$$2(-\frac{1}{2}) + 4y = 16$$

$$4y - 1 = 16$$

$$4y = 17$$

$$y = \frac{17}{4}$$

$$3(-\frac{1}{2}) + 2(\frac{17}{4}) = 7$$

(10)

$$-\frac{3}{2} + \frac{17}{2} = 7$$

$$\frac{14}{2} = 7$$

$$x = -\frac{1}{2}, \quad y = \frac{17}{4}$$

4.) Find determinant and inverse of A:

Find determinant:

$$A = \begin{bmatrix} 1 & 1 & 6 \\ 1 & 2 & 2 \\ 1 & 3 & -1 \end{bmatrix}$$

$$|A| = 1 \begin{vmatrix} 2 & 2 \\ 3 & -1 \end{vmatrix} - 1 \begin{vmatrix} 1 & 2 \\ 1 & -1 \end{vmatrix} + 6 \begin{vmatrix} 1 & 2 \\ 1 & 3 \end{vmatrix}$$

$$= (-2-6) - (-1-2) + 6(3-2)$$

$$= -8 + 3 + 6$$

$$\underline{\underline{|A| = 1}}$$

Find cofactor matrix:

$$A^c = \begin{bmatrix} -2-6 & -(-1-2) & 3-2 \\ -(-1-18) & -1-6 & -(3-1) \\ 2-12 & -(2-6) & 2-1 \end{bmatrix} = \begin{bmatrix} -8 & 3 & 1 \\ 19 & -7 & -2 \\ -10 & 4 & 1 \end{bmatrix}$$

Adjoint matrix:

$$\text{Adj}(A) = (\text{cof}(A))^T = \begin{bmatrix} -8 & 3 & 1 \\ 19 & -7 & -2 \\ -10 & 4 & 1 \end{bmatrix}^T$$

$$= \begin{bmatrix} -8 & 19 & -10 \\ 3 & -7 & 4 \\ 1 & -2 & 1 \end{bmatrix}$$

(10)

Inverse of A:

$$A^{-1} = \frac{\text{Adj}(A)}{\det(A)} = \frac{1}{1} \begin{bmatrix} -8 & 19 & -10 \\ 3 & -7 & 4 \\ 1 & -2 & 1 \end{bmatrix} = \begin{bmatrix} -8 & 19 & -10 \\ 3 & -7 & 4 \\ 1 & -2 & 1 \end{bmatrix}$$

5.)

$$\begin{bmatrix} 1 & 1 & 6 \\ 1 & 2 & 2 \\ 1 & 3 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = A^{-1} \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -8 & 19 & -10 \\ 3 & -7 & 4 \\ 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$$

$$\begin{array}{c|c} X & \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix} \\ \hline \begin{bmatrix} -8 & 19 & -10 \\ 3 & -7 & 4 \\ 1 & -2 & 1 \end{bmatrix} & \begin{bmatrix} -8+2 \times 19-40 \\ 3-14+16 \\ 1-4+4 \end{bmatrix} \end{array}$$

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$$\underline{\underline{\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -10 \\ 5 \\ 1 \end{bmatrix}}}$$

6.)

$$\frac{3x+2}{2x^2-x-1}$$

Factorise denominator

$$2x^2-x-1 = (2x+1)(x-1)$$

$$\frac{3x+2}{(2x+1)(x-1)} = \frac{A}{(2x+1)} + \frac{B}{(x-1)}$$

$$3x+2 = A(x-1) + B(2x+1)$$

When $x = -\frac{1}{2}$:

$$3(-\frac{1}{2}) + 2 = A(-\frac{3}{2}) + B(0)$$

$$\frac{1}{2} = -\frac{3}{2}A$$

$$1 = -3A$$

$$-\frac{1}{3} = A$$

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When $x = 1$:

$$3(1) + 2 = A(0) + B(3)$$

$$5 = 3B$$

$$\frac{5}{3} = B$$

$$\frac{3x+2}{2x^2-x-1} = \frac{-1}{3(2x+1)} + \frac{5}{3(x-1)}$$

~~Checking ans~~ Checking my answers (please ignore)

i.) i.) ~~$2x + 4y - 5z - 5x - 9$~~

$$\begin{array}{r} 2x + 4y - 5z \\ -5x - 9y + 2z \\ + \quad 4x - 7y + 8z \\ \hline x - 12y + 5z \end{array} \quad \checkmark$$

ii.) $x(x+2) - (x-4)$

$$x^2 + 2x - x + 4$$

$$x^2 + x + 4 \quad \checkmark$$

iii.) $3p^2qr \cdot 4pq^3 \cdot 5qr^2$

$$60p^3q^5r^3 \quad \checkmark$$

iv.) $\frac{6p^2q^3r}{3p^3q^3r^{-2}} = 2 \frac{r^3}{p} \quad \checkmark$

v.) $\frac{2fg}{16h} \times \frac{4gh^2}{4fh} \times \frac{32fh^3}{12f^3}$

$$\frac{256}{768} \left(\frac{f^2g^2h^5}{f^4h^2} \right)$$

$$\frac{1}{3} \left(\frac{g^2h^3}{f^2} \right)$$

$$\frac{g^2h^3}{3f^2} \quad \checkmark$$

$$2.) \quad i.) \quad 5(x-3) = 2(x+6)$$

$$5x - 15 = 2x + 12$$

$$3x = 27$$

$$\underline{\underline{x = 9}} \quad \checkmark$$

Checking answers
please ignore

$$ii.) \quad \frac{x+6}{2} = x + \frac{2x-5}{3}$$

$$3(x+6) = 6x + 2(2x-5)$$

$$3x + 18 = 6x + 4x - 10$$

$$28 = 7x$$

$$\underline{\underline{4 = x}} \quad \checkmark$$

$$3.) \quad \begin{aligned} 2x + 4y &= 16 \quad (1) \\ 3x + 2y &= 7 \quad (2) \end{aligned}$$

① * 2 ②:

$$2x + 4y = 16$$

$$\begin{array}{r} \cancel{2x} \\ - 6x + 4y = 14 \end{array}$$

$$-4x = 2$$

$$\underline{\underline{x = -\frac{1}{2}}}$$

$$3(-\frac{1}{2}) + 2y = 7$$

$$2y - \frac{3}{2} = 7$$

$$4y - 3 = 14$$

$$4y = 17$$

$$\underline{\underline{y = \frac{17}{4}}}$$

$$\cancel{-1 + 11 = 16}$$

4.) Find $|A|$, A^{-1} :

$$A = \begin{bmatrix} 1 & 1 & 6 \\ 1 & 2 & 2 \\ 1 & 3 & -1 \end{bmatrix}$$

checking answers
please ignore

$$|A| = \begin{vmatrix} 2 & 2 \\ 3 & -1 \end{vmatrix} - \begin{vmatrix} 1 & 2 \\ 1 & -1 \end{vmatrix} + 6 \begin{vmatrix} 1 & 2 \\ 1 & 3 \end{vmatrix}$$

$$= -2 - 6 - (-1 - 2) + 6(3 - 2)$$

$$= -8 + 3 + 6$$

$$= \underline{1}$$

$$\text{Cofactors}(A) = \begin{bmatrix} -2-6 & -(-1-2) & 3-2 \\ -(-1-18) & -1-6 & -(3-1) \\ 2-12 & -(2-6) & 2-1 \end{bmatrix} = \begin{bmatrix} -8 & 3 & 1 \\ 19 & -7 & -2 \\ -10 & 4 & 1 \end{bmatrix}$$

$$A^{-1} = \text{Cofac}(A)^T = \begin{bmatrix} -8 & 19 & -10 \\ 3 & -7 & 4 \\ 1 & -2 & 1 \end{bmatrix} \quad \checkmark \checkmark$$

5.)

$$\begin{pmatrix} 1 & 1 & 6 \\ 1 & 2 & 2 \\ 1 & 3 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -8 & 19 & -10 \\ 3 & -7 & 4 \\ 1 & -2 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}$$

x	$\begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}$
$\begin{pmatrix} -8 & 19 & -10 \\ 3 & -7 & 4 \\ 1 & -2 & 1 \end{pmatrix}$	$\begin{matrix} -8 + 2 \times 19 - 40 \\ 3 - 14 + 16 \\ 1 - 4 + 4 \end{matrix}$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -10 \\ 5 \\ 1 \end{pmatrix} \quad \checkmark \checkmark$$

$$\frac{3x+2}{2x^2-x-1} = \frac{-1}{3(2x+1)} + \frac{5}{3(x-1)}$$

6.) $\frac{3x+2}{2x^2-x-1} = \frac{3x+2}{(2x+1)(x-1)} = \frac{A}{2x+1} + \frac{B}{x-1}$

$$3x+2 = A(x-1) + B(2x+1)$$

When $x = -\frac{1}{2}$:

$$3\left(-\frac{1}{2}\right) + 2 = A\left(-\frac{3}{2}\right) + B(0)$$

$$\frac{1}{2} = -\frac{3A}{2}$$

$$\underline{-\frac{1}{3} = A}$$

When $x = 1$:

$$3+2 = A(0) + B(3)$$

$$5 = 3B$$

$$\underline{\frac{5}{3} = B}$$